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ADDENDUM

Volume **163**, Number 2 (2000), in the article "Effect of Symmetry to the Structure of Positive Solutions in Nonlinear Eliptic Problems," by Jaeyoung Byeon, pages 429–474 (doi:10.1006/jdeq.1999.3737): On p. 429, the title should read "In Nonlinear Elliptic Problems." In addition, the proof of the following lemma, stated on p. 452, requires revision:

LEMMA 4.5. *For a nonnegative minimizer u_R of $\Gamma_{R,M}$, it holds that, for any $\delta > 0$ and $x_0 \in M$,*

$$\lim_{R\to\infty} \int_{\Omega_R \setminus M_R^\delta} (u_R)^{p+1}\, dx = 0.$$

Correction of the Proof. Suppose that our claim is false. Then, for some $\lambda > 0$ and $\delta > 0$, there exist $\{R_i\}$ with $\lim_{i\to\infty} R_i = \infty$ such that

$$\int_{\{x \in \Omega_{R_i} \mid \operatorname{dist}(x, M_{R_i}) \geqslant \delta\}} (u_{R_i})^{p+1}\, dx \geqslant \lambda$$

for any $i = 1, 2, \ldots$. It is easy to see that $\delta < \delta_0$. Taking a subsequence of $\{i\}$ if necessary, we can assume that one of *vanishing*, *dichotomy*, or *compactness* in Proposition 1.2 occurs for the sequence $\{(u_{R_i})^{p+1} / \int_{\Omega_R} (u_{R_i})^{p+1}\, dx\}$. For convenience, we denote R_i by R. As we showed in the original proof of Lemma 4.5, vanishing or compactness cannot occur for the $\{(u_{R_i})^{p+1} / \int_{\Omega_R} (u_{R_i})^{p+1}\, dx\}$. Thus, *dichotomy* occurs.

In the original proof, we considered two minimization problems $J_R^\pm$. It is not certain that there exist minimizers of $J_R^\pm$. Since *dichotomy* occurs, we conclude in the same manner as in the original proof that there exist $\lambda^1, \ldots, \lambda^n \in (0, 1)$ with $\sum_{j=1}^n \lambda^j = 1$ such that for any $\varepsilon > 0$, there exist sequences $\{x_R^j\}_R$ for each $j = 1, \ldots, n$ and a constant $R_0 > 0$ with the property

$$\lim_{R\to\infty} \operatorname{dist}(x_R^i G, x_R^j G) = \infty \qquad \text{for} \quad i \neq j; \text{ and}$$

there exist $C > 0$ such that for all $R > R_0$,

$$\sum_{j=1}^n \left| \lambda^j \int_{\Omega_R} (u_R)^{p+1}\, dx - \int_{B_G(x_R^j, C)} (u_R)^{p+1}\, dx \right| \leqslant \varepsilon.$$

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Since the $\{\int_{\Omega_R} |\nabla u_R|^2\, dx\}$ is bounded, we can deduce, as in the case of *compactness*, that, if R is sufficiently large, then

$$\#(x_R^j G) < \infty \qquad \text{for} \quad j = 1, ..., n.$$

Redefining the sequences $\{x_R^j\}_R$, $j = 1, ..., n$ if necessary, we can assume that for some $g_k^j \in G$, $k = 1, ..., m_R(j)$,

$$B_G(x_R^j, C) = \bigcup_{k=1}^{m_R(j)} B(g_k^j x_R^j, C),$$

and that

$$\lim_{R\to\infty} |g_k^j x_R^j - g_{k'}^j x_R^j| = \infty \qquad \text{for} \quad k \neq k', \quad j = 1, ..., n,$$

where $m_R(j) \equiv \#(x_R^j G)$, that is, the number of elements of $x_R^j G$. Letting $d_R(x) = \min\{|x - x_R^j G|\ j = 1, ..., n\}$, from Lemma 4.2, we see that for some constants c, $C > 0$, $u(x) \leqslant C \exp(-c d_R(x))$. Moreover, it is easy to see from condition (A1) that $\liminf_{R\to\infty} \max_{x \in B(g_k^j x_R^j, C)} u_R(x) > 0$ for each j, k. We claim that there exist $y_k^j(R) \in \Omega_R$, $j = 1, ..., n$, $k = 1, ..., m_R(j)$ and $\eta > 0$ with the following property:

$$\begin{gathered}\limsup_{R\to\infty} |y_k^j(R) - g_k^j x_R^j| < \infty,\\ \liminf_{R\to\infty} \min\{u_R(x) \mid x \in B(y_k^j(R), \eta),\ i = 1, ..., n,\ j = 1, ..., m_R(j)\} > 0.\end{gathered} \tag{10}$$

Since we have

$$\int_{\Omega_R} \chi_R(u_R)^{p+1}\, dx \leqslant 1,$$

from an estimate [Theorem 8.25, GT], we see that for each $c > 0$,

$$\limsup_{R\to\infty} \max\{u_R(x) \mid x \in \Omega_R,\ \operatorname{dist}(x, M_R^\delta) \leqslant c\} = 0.$$

We note that

$$\Delta u_R + h u_R + f(u_R) + \frac{\alpha(R)}{1+\alpha(R)} (f'(u_R)\, u_R - f(u_R)) = 0 \qquad \text{on } M_R^\delta$$

and that $\limsup_{R\to\infty} \|u_R\|_{L^\infty} < \infty$. Then, from the elliptic estimates (refer to [GT]), we see that for each $c > 0$, $\limsup_{R\to\infty} |u_R|_{C^{1,\lambda}(M_R^\delta(c))} < \infty$, where $M_R^\delta(c) \equiv \{x \in \Omega_R \mid \operatorname{dist}(x, \complement M_R^\delta) \geqslant c\}$. Suppose that our claim (10) is not true. Then, for some $j \in \{1, ..., n\}$ and $k \in \{1, ..., m_R(j)\}$, there exist

$z_k^j(R) \in \partial M_R^\delta$ and $C > 0$ such that $\liminf_{R\to\infty} \int_{\Omega_R \cap B(z_k^j(R),\, C)} u^{p+1}\, dx > 0$, and that for each $c > 0$,

$$\limsup_{R\to\infty} \sup\{u_R(x) \mid \mathrm{dist}(x, \partial M_R^\delta) > c\} = 0.$$

Let $\lambda_1(R, C, c)$ be the first eigenvalue of $-\Delta$ on $B(z_k^j(R), C) \cap \{x \in \Omega_R \mid \mathrm{dist}(x, M_R^\delta) \leqslant c\}$ with Dirichlet boundary condition zero. It is known that $\lim_{c\to 0} \lambda_1(R, C, c) = \infty$ uniformly with respect to R (refer to [BNV, Lie]). Then, from Hölder's inequality and the Sobolev embedding theorem, we deduce that

$$\liminf_{R\to\infty} \int_{\Omega_R \cap B(z_k^j(R),\, C)} |\nabla u_R|^2\, dx = \infty.$$

This contradicts Lemma 4.3. Thus the claim (10) is true. Then, as in the original proof, applying Proposition 1.4, we conclude that

$$\liminf_{R\to\infty} \Gamma(u_R) > \#(xG)\, \Gamma^\infty, \qquad x \in M.$$

This contradicts Lemma 4.3. Thus, *dichotomy* cannot occur. This contradicts the result that one of *compactness*, *vanishing*, and *dichotomy* occurs for the sequence $\{(u_{R_i})^{p+1} / \int_{\Omega_R} (u_{R_i})^{p+1}\, dx\}$, and thus it holds that for any $\delta > 0$,

$$\lim_{R\to\infty} \int_{\Omega_R \setminus M_R^\delta} (u_R)^{p+1}\, dx = 0.$$

This completes the proof. ▮

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